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Psychological Pricing in Private Equity Transactions

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Abstract

Private Equity (PE) valuation is usually undertaken using traditional corporate finance models that do not consider psychological biases in such transactions. Such biases can have a significant impact on the final valuation and should be considered. Game theory and option pricing models can be used to incorporate such psychological biases in PE valuation. This paper develops a two-person PE valuation model using real options games to help understand if a Nash Equilibrium exists for such a transaction. This model indicates that the PE valuation may be quite different from what is prescribed by traditional models and the outcome will depend on the psychological biases of the PE investor (investor) and PE firm (investee).

Keywords: *Private Equity, Incomplete Games, Option Pricing*

Introduction

Private Equity (PE) transaction valuation is quite a challenging task in practice as psychological biases exist in such valuation. Traditionally, PE valuations are undertaken using traditional corporate finance models like the net present value and discount cash flow based approaches, for example, Rappaport (1998) or option pricing models, like Black and Scholes (1993) and Cox, Ross and Rubenstein (1979). However, none of these valuation methods incorporate psychological biases like risk-aversion or optimism in the model. Interestingly, such factors do impact PE valuation, for example, Cooney, Moeller and Stegemoller (2009) have found in their empirical analysis that PE valuations are impacted by psychological biases. Baker, Pan and Wurgler (2009) have also found evidence that psychological factors can impact valuations. Some of this evidence suggests that traditional finance models could be considered less accurate in valuing PE transactions.

In order to incorporate psychological biases in such transactions, researchers have used game theoretic models to undertake PE transaction valuation, for example, are Smit (2001, 2003) and Scholes et al. (2007). Other researchers have also applied game theory to solve valuation problems that are Farrell and Shapiro (1990), Hirshleifer and Ping (1990), Kinnunen (2010) and Yu and Xu (2011).

In this paper, we develop a two-person PE model using real options games, where we try to understand the strategy that the PE investor and PE firm will follow based on incomplete information. The structure of this real options game will be developed as follows:

1. First, a binomial tree (using the Cox, Ross and Rubinstein model) is setup to obtain the bid/ask and strike prices that the PE investor and PE firm will be willing to pay based on their type.

2. Next, a two-person incomplete information game will be developed with the pay-offs provided by the binomial model.

3. Finally, at each node in this game, the PE investor will offer the high or low bid/ask price in the binomial tree and the PE Firm will accept if it is a Nash Equilibrium point (i.e. it is the best offer it can obtain) and the game will end. Otherwise, the offer will be rejected and the game will move to the next higher node with the higher bid/ask price.

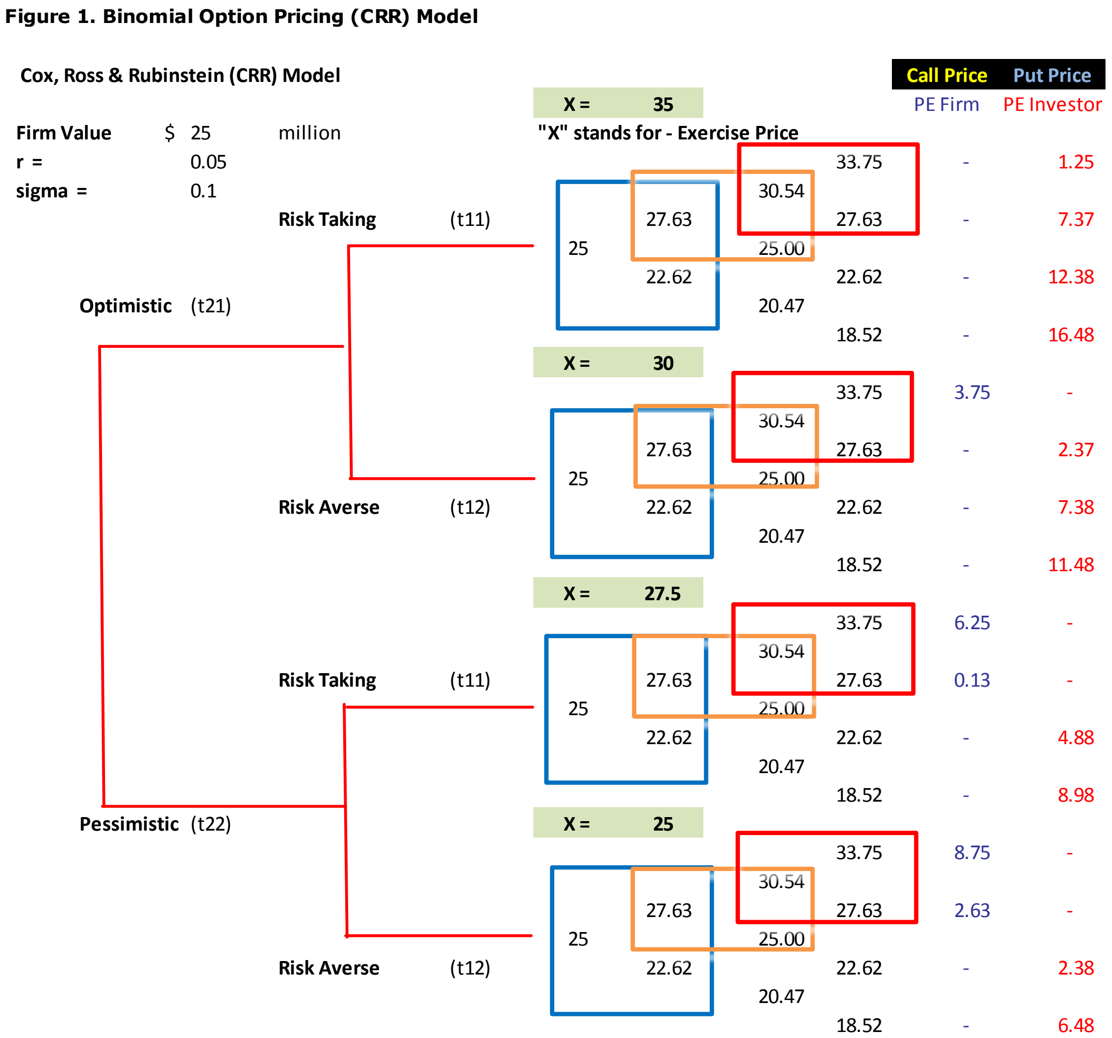
This paper is set out in the following structure: the next section will build the two-person Private Equity (PE) model. This model will also provide the Nash Equilibrium point of such a game. Finally, the last section will summarise the discussion that has occurred in this paper.

Two-person Private Equity (PE) Model

We develop a two-person Private Equity (PE) model that intends to identify the optimal strategy that the PE investor and PE firm should play in this real options game with incomplete information. Such deals in practice occur under the same conditions where neither the PE investor nor the PE firm know the personality or other information of the opponent. However, both want to find out what will be the strategy that will help each of them obtain the highest pay-off. To develop this two-person PE model, we need to start by setting up the binomial option pricing model that will provide the pay-offs for the incomplete information game (shown in figure 1 below). In figure 1, you will notice that the PE investor has the behavioural traits being risk-averse and risk-taking, while the PE firm has the traits being optimism and pessimism. The spot price in this game is $25 million and strike prices are $25, $27.5, $30 and $37.5 million.

You will also notice a column stating the call and put option prices in figure 1. As this binomial tree calculates an American option, the price of a call is calculated as max [0, spot – strike] and for a put it is max [0, strike – spot]. The call and put option prices that have been calculated in figure 1 and 2 align with the spot price in round four of this game. You will notice that these options can be out of the money in some cases, where their value is zero. The intent of developing this option pricing model is to find out which option will be viable and then to use these option prices in an incomplete game to find the Nash Equilibrium of such a game. This will explain the optimal strategy for the PE investor and PE firm in such a real options game.

Figure 1. Binomial Option Pricing (CRR) Model



Based on the binomial tree, the price of the option will either increase or decrease, which will result in the PE firm either accepting or rejecting this offer in view of the strike price of this option. If the offer is accepted then the game will conclude. Otherwise, the PE investor will increase the offer to the next higher node in the binomial tree - offer will not increase if the next higher node has a higher value than the strike price for that PE investor/PE firm type.

This can be seen in figure 2, where the offer price is increased by the PE investor in the instance where the strike price is $35 million and the maximum increase in spot price by the fourth round is $33.75 million. If the PE firm rejects this offer of $33.75 million in the fourth round then the value of the spot should be higher than $35 million. As, this is not viable for the PE investor the game will terminate in round four if the PE investor is risk-averse and the PE firm is optimistic. However, if the PE investor and PE firm have any other type that relate to the other strike prices of $25, $27.5 or $30 million, then you will notice the game will terminate in round one. The yellow squares in figure 2 provide the path of the offers that the PE investor will provide.

Figure 2. Offer Path in the Binomial Option Pricing (CRR) Model

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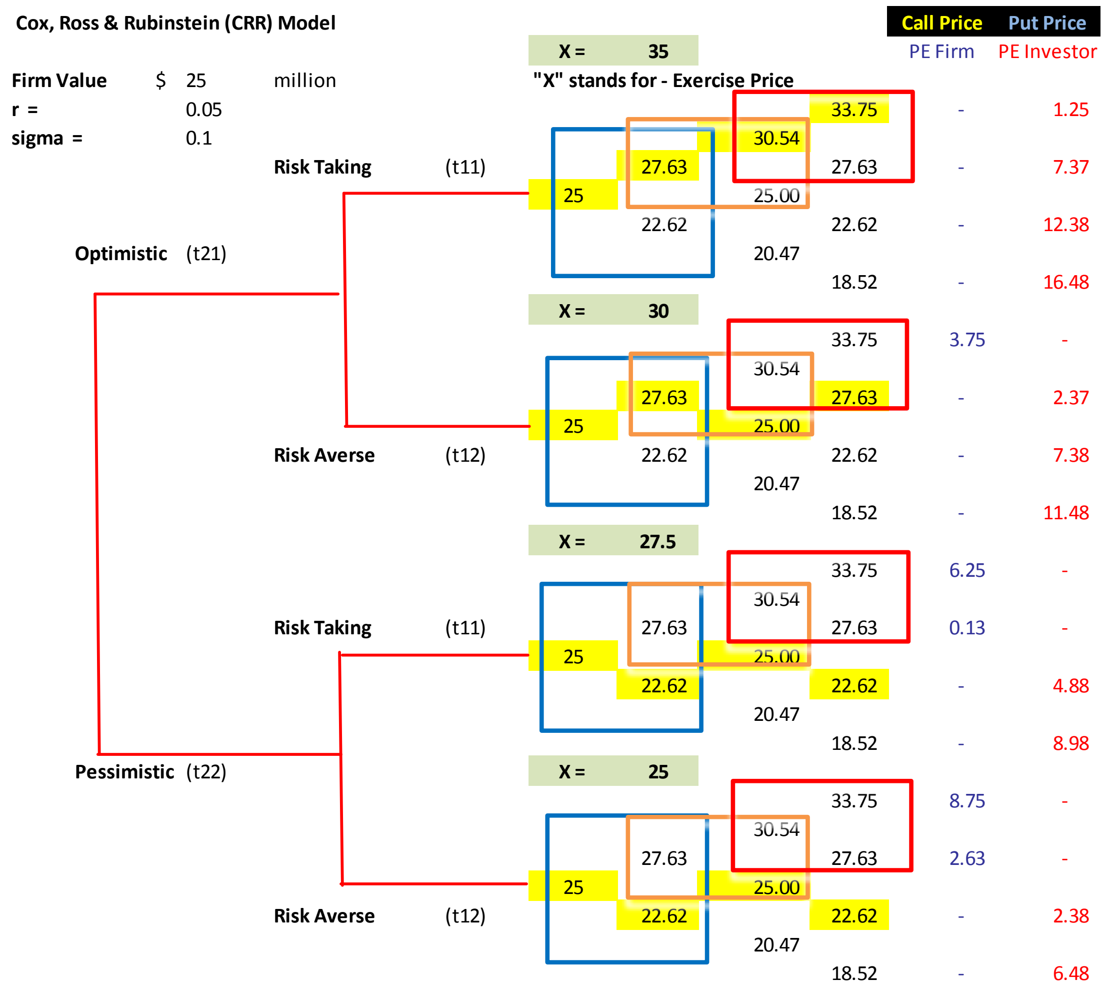

Based on the binomial option pricing tree that has been developed, we can associate them to the two strategies that can be played by the PE investor and PE firm. These strategies are identified in figure 3.

Figure 3. Player Strategies and Option Pricing

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Acquirer's Strategies

a1 = Increase Bid

a2 = Decrease Bid

Target's Strategies

b1 = Accept

b2 = Reject (if Reject, then moves to next Game - PE Investor Increases Bid)

Option Type

PE Firm - Holds (Short Put) a Call Option - As he will gain from the price **increase** (Obtain a higher price for the sale)

PE Investor - Holds (Short Call) a Put Option - As he will gain from the price **decrease** (Pay less to buy the PE Firm)

Strike/Exercise Price - Amount the PE Investor is expected to Offer & Price the PE Firm will likely Accept - based on each **Type**

These strategies are associated with the call and put option prices that are provided in figures 1 and 2. In order to find the pay-offs for these strategies for the PE investor and PE firm, we first need to state the probabilities associated with these strategies in this incomplete information game - these probabilities were taken as an example and can be modified based on the probabilities related to each player's type, shown in figure 4.

Figure 4. Probabilities based on player type

P(t12,t21)	0.2	**P(t12,22)**	0.3
P1(t21|t11)	0.80	P1(t22|t11)	0.20
P1(t21|t12)	0.40	P1(t22|t12)	0.60
P2(t11|t21)	0.67	P2(t12|t21)	0.33
P2(t11|t22)	0.25	P2(t12|t22)	0.75

Once we consider the probabilities associated to the type and strategy of each player, we are able to calculate the pay-offs that each player will receive for playing these strategies. These pay-offs are provided in figure 5.

In figure 5, we can see that the optimal strategy that the PE investor can play will be **a2a2** and the PE firm will play the strategy **b1b1**. This will result in the PE investor gaining more than the PE firm in this game. The reason that the PE investor will decrease is that the PE investor does not want to pay anymore than required to purchase the PE firm and the PE firm may accept the offer because there is a one in four chance that the PE investor is risk-taking while the PE firm is optimistic.

Figure 5. Pay-offs for each player for playing specific strategies

Figure 5. Pay-offs for each player for playing specific strategies

PE Investor's Pay-off Profile

	b1	b2	
a1	7.37	-	Type = (t11,t21)
a2	12.38	-	

	b1	b2	
a1	2.37	-	Type = (t11,t22)
a2	7.38	-	

	b1	b2	
a1	-	-	Type = (t12,t21)
a2	4.88	-	

	b1	b2	
a1	-	-	Type = (t12,t22)
a2	2.38	-	

PE Firm's Pay-off Profile

	b1	b2	
a1	2.63	-	Type = (t11,t21)
a2	-	-	

	b1	b2	
a1	2.63	-	Type = (t11,t22)
a2	-	-	

	b1	b2	
a1	2.63	-	Type = (t12,t21)
a2	-	-	

	b1	b2	
a1	2.63	-	Type = (t12,t22)
a2	-	-	

PE Investor's Optimal Strategy

	b1b1	b1b2	b2b1	b2b2
a1a1	3.1854	3.1854	-	-
a1a2	4.8749	3.1854	1.6895	-
a2a1	5.6895	5.6895	-	-
a2a2	7.3791	5.6895	1.6895	-

PE Firm's Optimal Strategy

	b1b1	b1b2	b2b1	b2b2
a1a1	2.6293	1.3146	1.3146	-
a1a2	1.3146	1.3146	-	-
a2a1	1.3146	-	1.3146	-
a2a2	-	-	-	-

Nash Equilibrium - PE Investor/PE Firm

	b1b1		b1b2		b2b1		b2b2	
	PE Investor	PE Firm	PE Investor	PE Firm	PE Investor	PE Firm	PE Investor	PE Firm
a1a1	3.1854	2.6293	3.1854	1.3146	-	1.3146	-	-
a1a2	4.8749	1.3146	3.1854	1.3146	1.6895	-	-	-
a2a1	5.6895	1.3146	5.6895	-	-	1.3146	-	-
a2a2	7.3791	-	5.6895	-	1.6895	-	-	-

Conclusion

In conclusion, the intention of this paper was to provide the two-person PE model as a potential framework that could be used to incorporate psychological pricing in PE transactions using real options games. In most cases, PE transactions have a lot of uncertainty surrounding the pricing of the PE firm and it becomes perplexing for the PE investor if the offer should be increased and for the PE firm as to what offer would be sufficient without rejecting any potential PE transaction. This paper intends to capture the incomplete information and explain how the PE investor and PE firm can understand what optimal strategy they can play in such a game.

As a result, this paper answers these key questions, where the option pricing model explains the highest offer price that an PE investor would offer in based on the different types of the PE investor and PE firm. This should explain to the PE investor and PE firm, as to the boundaries of the potential sale price that may eventuate. However, more interestingly, it is the type of the PE investor and PE firm that really determines the outcome. In an incomplete information game, we do not know the type of the opponent. However, based on the probability of each type we can make some assessment on the likelihood of the strategies and offers that the opponent will provide, which should provide us some understanding of the strategies that we should follow to obtain the best outcome. For example, if the PE investor has a higher probability of being risk-taking, it makes sense for the PE firm to act optimistically and demand a higher price. On the contrary, if the PE investor finds the PE firm to be pessimistic, then he should act as if he is risk-averse by pushing down the price.

Therefore, we realise that behaviour of each player can be a strong indicator on what strategies are followed and as a result the final outcome of the PE transaction. It is often seen that PE investors pay more than they should have paid to take over the PE firm. So, it may useful to understand that it is not about how much more you can pay to take over a PE firm, but to understand the dynamics of the game and the behaviour of the players in the game. This will help both the PE investor and the PE firm optimise their outcome for a PE transaction.

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