On Teaching Engineering Mathematics

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Abstract

The fact that mathematics is an indispensable part of the curriculum of any engineering course goes without saying. But, as everyone involved in the teaching of the subject can testify, making it relevant, or at least appear relevant, to the needs of undergraduate engineers is far from the easiest of tasks. In addition, some students poorly relate the mathematics they are taught in one course to that which is applicable to their core engineering subjects. It may also be the case nowadays, particularly in the era of computer-algebra and sophisticated simulation software, that both the knowledge and the types of mathematical skills actually required by engineers may have changed significantly and that the curricula and teaching practice of many engineering-mathematics courses may not yet reflect this new reality.

This paper explores issues relating to the effective teaching (and learning) of mathematics for engineers; this includes: course content, teaching methods, as well as possible ways of inspiring young engineers to see mathematics as a useful friend rather than a necessary but somewhat annoying, companion. It also considers questions such as: what mathematical skills are now essential in order to be a competent engineer, and when does the mathematics presented need be proved rigorously or when will heuristic or less formal proofs suffice? In addition to presenting some insights based on the author's experience of teaching undergraduate mathematics, engineering and physics courses, the paper also attempts to address some of the difficulties, challenges and shortcomings of the traditional teaching of the subject and offers some new perspectives and suggestions.

Keywords: *engineering; mathematics; education.*

1. Introduction

There is a world-wide trend towards increasing the number of young people entering higher education. In other words, more students than ever are going to university to study, and this puts even greater pressure on those who provide service mathematics courses (Taylor, Mander, 2002). In addition, there is often a considerable range in the mathematical abilities and knowledge of students arriving at universities to study various engineering courses. Most engineering degree courses use the first year to ensure that all students have a firm grounding in basic subjects, particularly mathematics and physics. The first year is also used to rectify, as best as possible, shortcomings or gaps in students' knowledge as a result of their disparate educational backgrounds and standards. However, it appears that some students are beginning engineering and science courses with less than the required or minimum level of mathematical skills for the intended level of study. The London Mathematical Society (2005) reported that in the UK there is "the perception of a marked change in mathematical preparedness even amongst the very best applicants". The reasons for this are clearly not because of any innate lack of ability on the part of the students, but primarily because of a variety of factors that are largely outside their control. For example, the foregoing reference notes the shift away in schools in England from core mathematical techniques to more time-consuming activities such as data surveys and investigations. A reduction in the amount of time (around 20%) spent teaching mathematics in schools, as well as a delay in the teaching or even the removal of “difficult” topics was also noted. Other factors at play include changes in school mathematics curricula without the necessary changes in university curricula (Croft, 2006), as well as “grade inflation” (BBC Online, 2004) (a euphemism, some may argue, for the lowering of standards). The fact that students may not have been always taught the subject by a qualified expert in the subject, especially at critical stages in their education is surely also an important issue. The shortage of qualified or competent mathematics (and physics) teachers in schools in comparison to other subjects is a widespread problem (Smith, 2006).

It may not be entirely correct to discuss problems, and possible causes, originating in one part of the world and presume that they apply elsewhere. Nevertheless, some, if not all, of the issues mentioned above will surely have a global resonance. Certainly, from my recent experience in teaching engineering mathematics and physics in Thailand, there appears to be a problem in the level of mathematical preparedness of some students entering university to study engineering. In fact, it could be argued that there is a problem with attitude towards mathematics per se on a much deeper level within society as a whole. Mathematics, or at least mathematical education, has something of an image problem. And apart from being seen as difficult (which it is relatively speaking) it is also often (incorrectly) perceived as being practically useless. In this context, I can recall once having a conversation with an experienced engineer who, surprisingly to me, was also quite skeptical about the significance of the rôle of much of higher mathematics in engineering. I have also witnessed fellow academics from other faculties “selling” their subjects to prospective students by promoting the fact that courses they offer involve working with computers whilst avoiding the necessity of doing mathematics. This apparent widespread “resistance” to mathematics is worrisome to say the least.

2. How best to teach engineering mathematics?

It surely cannot be disputed that mathematics for engineers and scientists should be taught in a way that is as integrated as far as possible with the subjects to which it is to be applied. Traditionally, however, the bulk of mathematics for engineering courses has been taught separately as part of service mathematics courses. Often, common service courses are offered for engineering students following a variety of disciplines such as electrical, mechanical, civil, software and chemical engineering. This is usually concentrated into the first two years of study (a fact that may itself cause some problems) though some specialized mathematics courses may be taught in the latter years of an engineering degree course.

In addition to teaching subjects in mathematics that may not appear to be immediately relevant to students’ needs, there is also the issue of the length of time between introduction to some mathematical ideas or techniques and their demonstrated application to engineering problems. It is also not uncommon to be confronted with a situation where the “horse has been put before the cart”, and students are exposed to an engineering concept without yet having been taught the relevant mathematics. One example that springs to mind from personal experience is when I have tried to use some concepts involving vectors in mechanics to students who have not yet formally met them, or at least not sufficiently rigorously, in their service mathematics courses. Being taught the same thing twice is the inevitable result, though this is not always necessarily a bad thing for many students, even if on paper it does not appear to be the most efficient use of teaching time. In an ideal world, the necessary mathematics would of course be taught “just in time”; but this is not a realistic or practicable prospect in most cases.

Various solutions have been suggested to address some of the concerns given above. Discipline-specific supplements to a core common engineering mathematics course are one possible way to solve, or at least ameliorate, the situation (Carpenter et al., 2001). A greater integration of the relevant mathematics with the engineering subjects, in other words, transferring a substantial part of the mathematics taught from separate service modules into the engineering modules themselves is another suggestion (Niklasson, 2002). For example, an undergraduate chemical engineering course concerned with rates of reaction might teach coupled ordinary differential equations in addition to the relevant chemistry. In this way, students immediately see the relevance of what they have been taught, and the examples they are given will be more appropriate than the more general or possibly even contrived or unfamiliar examples given to them in a general service mathematics course.

Some mathematics educators have argued that engineering mathematics should be taught by engineers and not at all by mathematicians. In a report entitled "*Engineering Mathematics Should be Taught by Engineers*" (Ahmad et al., 2001), the authors make the case that engineers would make better teachers of engineering mathematics because they are "unlikely to become entangled with providing unnecessary proofs", and will not provide "unrealistic examples". The authors of the same report also suggest that engineering lecturers can "better relate" to their students. (The issue of "unnecessary proofs" will be considered here later). Improving the realism of the mathematics taught to students is a key issue in the teaching of mathematics to engineers and scientists. The third point made by the authors, namely of having empathy with the students is also important. It is natural to assume that an experienced mechanical engineer, for example, would be more attuned to the needs of mechanical engineering students than a person who exclusively thought of themselves as a professional mathematician. It is relevant, however, to point out here that there are also those somewhat peculiar animals who think of themselves as being exclusively engineering mathematicians. And surely an effective engineering mathematician will make it a priority to empathize with the needs of their students, teaching them what is needed and emphasizing what is especially important and applicable rather than blindly teaching the subject formulaically for its own sake. There is surely a strong case for saying, as is the case with all academic disciplines, that the most effective teaching is delivered by specialists who are cognizant of the needs of the students and who are broadly familiar with the courses they are pursuing. As a case in point, poor or ineffective mathematics teaching in schools is often said to occur exactly because it is not carried out by specialists, or even by people who especially enjoy or respect the subject. The teaching by non-specialists is often cited as a major reason why some students remain weak in mathematics and in turn view the subject with some misgivings or displeasure. With regard to university-level teaching of the subject, specifically in the context of engineering mathematics taught as a service course, the syllabus of a particular course should have been developed in cooperation with the various engineering schools and should reflect their needs rather than just representing the way things have always been done in the past or the particular interests or expertise of the engineering mathematics lecturers who will teach the course.

In assessing the best ways to help students through effective teaching, it is also relevant to think about the best way students can be helped or motivated to teach themselves and to continue the learning process outside the lecture theatre. As a high-school pupil, I still recall hearing one of my teachers saying that mathematics was different from other subjects in that you only really learned it by actually doing it yourself rather than being told what to do. Arguably, to some extent this is true of all technical subjects, but it does seem to be especially true of mathematics where understanding and the development of objective problem-solving skills clearly trumps memorization and rote learning. Indeed, the most effective way to achieve a deeper understanding of a mathematical technique or concept is to first judge that it is appropriate for the problem at hand, having possibly selected it from a number of alternatives and having adapted it, if necessary, into a suitable form. The ability to apply newly-learned mathematical knowledge and to combine it with previously-learned material represents an ideal learning scenario.

When providing assistance to students solving problems, it is generally accepted that tutors should only give guidance when it is felt that the student has genuinely made some effort to "break into" the problem. Naturally, many students when presented with new problems will first look for some correspondence between the one they are currently faced with and the solution of an example previously given to them by their lecturer. If a linkage is not immediately forthcoming, disillusionment may set in only to be followed by the familiar plea for help: "How do you do this problem, sir?" An analogy with spoon feeding is very appropriate in this situation. Students that overly rely on immediate assistance fail to develop the necessary confidence that should be part of their educative process. Moreover, they will fail to develop the necessary tenacity needed later to solve problems involving the application of mathematics to real life situations. Ideally, students will have already attempted and also had time to think about the more demanding problems before attending a scheduled tutorial session. Giving students enough time to think by not making excessive and unrealistic demands on both their time and capabilities is also important in developing their confidence in mathematics and in furnishing their storehouse of mathematical skills.

Occasionally, when teaching engineering mathematics, it is absolutely necessary to state what might seem to be the obvious. What may seem self-evident or manifestly true to someone who has been teaching or practicing the subject for years will not always be so for a student meeting something unfamiliar for the first time. Students occasionally get held back in their understanding just for want of a simple, even trivial, additional explanation or clarification; this also applies to textbooks and lecture notes where students are sometimes frustrated by the omission of a key step in a proof, a derivation or a solution. Aside from any desire to be economical, critical omissions appear to happen because they appear to be obvious or transparent to the authors and hence their inclusion is deemed superfluous. For example, looking at the customer reviews of a revised edition of one famous engineering mathematics textbook (on the commercial Amazon booksellers site on the worldwide web) I came across the following comment: "… I think they've packed too much in sometimes, without covering it properly, or just by omitting things because they are supposedly 'obvious'; well not to everyone …" ("Disappointing" {by 'Rumplestiltskin'}, 2006). It should be also stated, however, that the reviewer does go on to indicate that the book in question is a good mathematics textbook, and that the complaint is motivated by how things have been omitted in comparison to an older edition of the same book.

3. The rôle of informal methods in teaching engineering mathematics

Most of us will already be familiar with informal, and sometimes irreverent, self-teaching guides for various subjects, typified by the likes of the Dummy series of books (Ryan, 2003). In addition to mathematics books for "dummies", there are also explanatory texts for "idiots" as well as for the "clueless". Though not wishing to offend, I am not quite sure of the relative ranking of "dummies", "idiots" or "the clueless"! Now, as someone who has used some of these books to bridge gaps in my computing knowledge, I can personally vouch for the effectiveness of such an approach, not to mention their entertainment value. I think these books are appealing and refreshing in their unpretentious and honest approach to instruction, and do have a vital role to play. Another famous example, though by no means one for dummies, idiots or the clueless that employs an informal and engaging didactic style, is the Feynmann Lectures on Physics series (Feynmann et al., 1977). Reading Feynmann's Lectures on Physics, with its mix of informality, acute perception and somewhat folksy language is a bit like having the crux of a difficult lecture explained by an intelligent and knowledgeable friend or fellow student. I think that the use of informal, and even entertaining or humorous, language can be very effective tool in teaching mathematics to engineers, as well as in making the subject generally less inhibiting.

Most educators can attest from personal experience that the injection of informality and humor into the teaching may have a very beneficial impact on students. Moreover, some self-deprecation and humility on the part of the lecturer can also help boost students' confidence and can demonstrate that the subject is less recondite and more accessible than it might otherwise appear to be. For example, if a lecturer owns up to the occasional error in a calculation or derivation, or even confesses to having once experienced difficulties with some advanced concepts, then this can reassure students that it is normal to have experiences like this when doing mathematics at the level they are being taught. Students may then appreciate that they should not become unnecessarily disheartened when things are difficult and challenging, or when things seem to be going wrong. Moreover, since mathematics is sometimes seen as a dry and unexciting subject, albeit an occasionally useful one by those not studying it as their main course of study, any informality that makes it more accessible and even entertaining is surely a good thing. Indeed, when discussing the formative years of James Clerk Maxwell, Hutchinson (2006) (Head of the Department of Nuclear Science at M.I.T.) states that: "Undoubtedly his father's patient informal tutoring was an abiding formative influence". In other words, patient, informal tutoring was an "*abiding formative influence*" of arguably the greatest engineering mathematician of modern times. This is surely something significant; especially taking into account the knowledge that Maxwell was no child prodigy, but rather someone who gradually developed his formidable analytical mathematical skills and insights over time.

It is also important to develop the habit of exploratory or heuristic thinking when solving problems in mathematics. An immediate example that comes to mind is in the simple harmonic motion equation: $x'' + \omega^2 x = 0$. Usually, from experience when first-year students are told that one can see, or "guess", that the solution to this differential equation is: $x = A\cos\omega t + B\sin\omega t$, many of them are initially distinctly unimpressed or unconvinced by this assertion. This is the case even after the insightful "guess" can be shown to be correct by direct substitution into the equation, where doubts still linger about how the solution was obtained in the first place; it is somehow felt by students that this is not a valid or proper way of doing mathematics as it does not seem to follow the familiar pattern of one thing leading to another, and on the surface does not appear to be formal enough. Maybe it is even seen something of a cheat, and that some prior step should by rights have been carried out to obtain the solution rather than apparently just pulling it out of thin air, albeit with the apparent assistance of insight. A similar reaction is often also encountered when integrating functions "by inspection", even when the guess can be verified, or modified slightly, by back differentiation.

In considering the issues surrounding formality and informality in the teaching of engineering mathematics, the thorny issue of providing proofs inevitably arises. It is clearly not possible to prove every single result rigorously. But on the other hand, merely just stating a radically new result and then fobbing students off with the old chestnut: "It can be shown" is also clearly not entirely satisfactory. Whilst the scenario where all results are rigorously proven would be overwhelming and impractical for both students and staff, the diametrically opposite approach of just stating results without any or much justification would surely engender rote memorization and uncritical thinking, and would not be beneficial to the attainment of essential logical and mathematical skills. Nevertheless, I do not think that teachers of engineering mathematics should be afraid of using less formal or heuristic proofs to justify results, even if the justification is only partial and not completely general. Indeed, such approaches are often more natural, and sometimes correspond to the insights that lead to the discovery of certain results in the first place. An informal or heuristic justification can also plant the seeds that will lead to a deeper understanding and a more formal validation later on. Intuition and insight have often been strong forces for breakthroughs and advances in mathematics, physics, chemistry, biology and science in general throughout human history.

With regard to the issue of providing a convincing mathematical proof for a given result, I can recall reading the following quote (but sadly I do not have a reference for it) said of a professor to his graduate student when presented with a mathematical proof: "Don't just prove it to me, convince me". The sentiment of this statement is very pertinent to the case of proving the validity of mathematical results to many engineering and science students. Sometimes, an informal proof is the most effective way to proceed before (if necessary) finally offering a more formal one. The informal explanation often allows students to appreciate the validity of a result in a way that a formal proof might obscure.

Some other issues to consider here are the actual reasons for giving proofs in engineering and service mathematics courses and the extent to which they are needed. As pointed out by Rowland (2000), the rôle of proof in standard scientific research and the rôle of proof in the classroom is quite different. In research, its purpose is for the assurance of truth, whilst in the context of teaching the primary purpose of proof is to *explain.* Moreover, the final emphasis in engineering mathematics is always going to be on the correct application of results. The extent to which mathematical proofs are needed (and their level of formality) is ultimately a matter of judgment taking into account their rôle as a tool for explanation and what can be assumed to be axiomatic when teaching new material.

4. Teaching resources

In the past, many of the available engineering-mathematics textbooks were undoubtedly dry, uninspiring and somewhat daunting tomes. Some even appeared to have been written specifically for people who already knew the subject rather than for genuine learners. But nowadays there are many excellent engineering-mathematics textbooks available that are comprehensive in scope and accessible, not to mention appealing, to students. Some have gained universal acceptance as standard or classic texts by lecturers; (e.g. Kreyszig, 1999). Such books are genuinely useful to students as a supplement to their lecture notes, as a source of useful exercises and also for self-study. As well as standard format text books, with explanations, examples and exercises, programmed-learning engineering mathematics textbooks can also be extremely effective, especially for students struggling with particular concepts. Readers are encouraged to work through the various steps in a problem and, depending on their answers at various stages, are directed onwards to the next stage or backwards to review and redo earlier material. One outstanding example of programmed learning in engineering mathematics is the universally-praised: “Engineering Mathematics” by Stroud (2001). I have used earlier editions of this book in the past, but was interested to read what present-day engineering-mathematics students think of this book. One customer review of the most recent version of the book on the Amazon book site typifies the praise that has been heaped on the various editions of this book: “*My God! What a revelation, for the first time here was a maths text book that helped me to understand exactly what was going on and took me step by step through increasingly difficult problems.*” (“This book saved my life!” {by ‘Mark’}, 2006).

Mathematics books of whatever form are enhanced in their effectiveness by accompanying material on CD ROMs or by providing access to a companion website for both staff and students. Most modern engineering mathematics textbooks provide at least one of these two options. There is also an increasing number of websites and forums devoted to all aspects of mathematics and physics. Students, researchers or the general public can post questions or problems on these forums, and provided the question or problem is sufficiently well-posed, a solution to the problem, or at least a helpful comment, is usually forthcoming. In the past, a student who was unable to get enough help from a lecturer or tutor for an assignment might turn to a fellow classmate for help. This is, of course, not the only way in which the internet can be used as an educational tool. There are many websites and resources available on the internet devoted to every aspect of mathematics and at all levels. As with so many other things, the internet will, or already has, revolutionized the teaching of engineering mathematics.

The availability of computer-algebra software packages is another example of a revolutionary development in the teaching of engineering mathematics. It offers new and far-reaching possibilities for the teaching of the subject. It is understandable that there might be some slight reservations about such a tool, much in the same way there were about the use of electronic calculators in school mathematics teaching, specifically in tests or exams, when they first appeared around the mid 1970s. The feeling that it is detrimental for students to be helped with numerical calculations, and now also with algebraic manipulations and operations in calculus, is clearly motivated by a fear that students relying on such tools will not develop essential skills or gain necessary insights or understanding of the subject. In the case of computer-algebra packages, there might also be the feeling that students will be able to obtain answers by shortcutting the usual steps needed in order to obtain solutions to problems. In other words, students may be able to obtain correct answers without necessarily fully comprehending how they arrived at them. A prime example of this is in calculus where functions can be symbolically differentiated, or integrated, merely by highlighting a particular variable in a term in a spreadsheet and selecting a menu option in the software. Some people may ask: Why fret over the level of proficiency a student (and later on perhaps, a practicing engineer) has in applying, for example, the quotient rule or the chain rule to a function when a piece of software can shoulder the burden? Moreover, as long as the general concept of a derivative is clearly understood, surely that is good enough. In response to this, others might say within reasonable limits that this view is correct, but that concepts still have to be properly understood, and that familiarity and proficiency with the basic results is what really assists in the development of proper understanding. Lessons can still be structured so that during problem-solving exercises, computer-algebra software is used to assist with the more mundane tasks as well in checking the correctness of final results. Furthermore, as a result of computer-algebra software, there is now greater scope for giving realistic problems to be solved, allowing a more effective use of the time available for learning. Most computer-algebra software packages contain programming features that are relatively easy to master (certainly in comparison to old programming languages such as FORTRAN), with clear and almost self-evident syntax, dispensing of the need to spend (i.e. waste) fruitless hours “debugging” as well as learning the syntax and idiosyncrasies of a particular language, such as was the case with the old technical programming languages: activities, which I know from personal experience used to be the bane of engineering mathematicians in the past. The new opportunities offered by this technical revolution should, in principle, lead to students gaining a deeper understanding of the mathematics much more efficiently and effectively. As an example, students will be able to quickly graph and then interpret or check results; they will also be able to explore with ease, for example, the effects of changing parameter values.

With sufficient imagination, the possibilities for the effective use of computer algebra software in engineering-mathematics teaching are boundless. Moreover, this is actually the way engineering problems are increasingly being solved now in real life, and will continue to be so in the foreseeable future. Computer-based tools are now seen as indispensable to the engineer as slide rules once were half-a-century or so ago. Common sense and judgment still, however, need to be used in applying new technology to mathematics teaching, especially with regard to the use of software.

Another issue in the effective application of computer-algebra software for the teaching of engineering mathematics, especially with first-year students, is that of being able to convince the students to actually make use of the software in the first place. Many students are initially skeptical of the benefits, and see it as just one extra thing they have to learn rather than something that might actually make their lives easier. Rielly (2004) points out that: “familiarity with the software is initially a barrier to student success in solving first-year problems, and that very few students have any experience in writing software input commands in any language or for any application”.

5. What essential skills, concepts and techniques need to be taught and encouraged?

The development of a general fluency in applying mathematical-reasoning in conjunction with engineering insight is a key aim of all engineering-mathematics courses. Some other important skills that students need to acquire related to the construction of adequate mathematical models is the ability to make reasoned assumptions as well as the ability to make sensible approximations. Developing general mathematical modeling skills is very important given that specific techniques or methods learned at university may later in practice have been superceded by newer techniques more in tune with the capabilities of available computer technology. An analogy would be in computer science where the development of the necessary logical skills that allow, for example, a computer programme to be written should take priority over knowing in detail the specifics of a given computer language, which may anyway eventually change or even become obsolete.

There are certain concepts that reoccur repeatedly in many different areas of engineering mathematics. For this reason, they are important, and need to be taught as early as possible, and re-emphasized as frequently as possible. When writing this paper, I thought about the different techniques and results from mathematics that I have actually used in research and in practical applications to industrial problems. I concluded that the most useful thing, in the sense that I had needed to use it the most often, was the ability to expand functions using one or multi-dimensional Taylor-series expansions, and to use them to make (more often than not) linear approximations. The linearization of non-linear expressions and functions was an activity whose necessity seemed to be almost ubiquitous in the application of mathematics to engineering problems. Indeed, throughout engineering and physics, non-linear systems are often linearized in order to obtain expressions valid for small perturbations about some nominal point. A familiar example of this occurs in high-school physics when analyzing the motion of a simple pendulum for small angular displacements about the vertical equilibrium position. The ever-presence of linearization (i.e. using the first-order terms in a Taylor series expansion of a given function) coupled with the assumption that it represents a valid approximation or assumption is the reason why it is often jokingly said that the first law of engineering is: "*All systems are linear!*". General familiarity with series expansions is essential in many areas of engineering (e.g. dynamics, control theory), as well as providing the theoretical basis for applied optimization, numerical integration, differential calculus, etc. etc.

I also think it is important that students are familiar with the concept of a system, and understand how to define a system and its boundaries early on in their studies of engineering mathematics. Gaining an understanding of some of the more difficult or potentially confusing results in, for example, mechanics, fluid mechanics or thermodynamics, is made easier if the general idea of how a system is defined and how it interacts with external influences across its boundaries has already been grasped.

One more concept involving systems that I believe is crucial in applying mathematics successfully to engineering is system identification together with the associated concept of parameter estimation. Traditionally, system identification involves finding both an adequate structure for a mathematical model (in other words, the general form of the equations to be used) as well as estimating the specific values of the coefficients or parameters within the model (i.e. the parameter estimation).

To establish what actually constitutes an adequate model, it is important to appreciate that in most practical cases there will be no such thing as a "true" model. The desired outcome is to obtain a model that is sufficiently adequate for its intended purpose. It is important for students to appreciate that all practical mathematical models are based on assumptions that determine the limits of their validity. The myth of the "true" model is still, however, very strong even amongst practicing engineers. Maine and Iliff (1985) note that, "*A favorite form of lunacy among aeronautical engineers produces countless attempts to decide what differential equation governs the motion of some physical object, such as a helicopter rotor… But arguments about which differential equation represents truth, together with their fitting calculations, are wasted time.*"

There is an important principle in mathematical modeling known as the "principle of parsimony", which states that any mathematical model should be as simple as possible; this is really just another way of expressing a basic guiding principle in philosophy, first suggested by the 14^{th}-century philosopher, William of Occam, known as Occam's razor ("Occam's Razor, Wikipedia Online, n.d.); this states that the simplest explanation of any phenomenon, one making the least assumptions, is generally speaking the best one. As a general rule, simple mathematical models are preferred to over-wrought or over-parameterized models that may have a satisfyingly close fit to any date used to obtain them in the first place, but which will have significantly poorer predictive qualities when used with different data.

Compiling a complete list of all the other essential skills that might be considered necessary for students studying engineering mathematics would probably just end up look like a copy of most engineering-mathematics syllabi. Therefore, I will avoid doing this and just mention five more things I consider are the *sine qua nons* of an engineering-mathematics education; these are: a complete understanding of the nature of a function; total competence in algebraic manipulation; familiarity with the general concept of using transforms (e.g. Laplace, Fourier, and Z-transforms) to convert a problem involving derivatives into one having a more convenient algebraic form; confidence in the use of dimensional analysis to validate expressions and to predict the forms of relationships; and, finally, a firm grounding in probability and statistics, especially probabilistic distributions with a view to their use in areas such as reliability analysis, error analysis, control-system design or optimal estimation.

6. Effective testing and grading of engineering mathematics

The use of examinations as a tool for assessment is clearly more problematic in some areas of engineering mathematics than in others. For example, much of the important numerical analysis in an engineering-mathematics course does not lend itself readily to the necessary time constraints of the traditional examination format. As a result, this may distort the teaching process in favor of more examinable but not necessarily more important aspects of a course. For example, lengthy and time-consuming numerical-analysis problems, such as the numerical solutions to partial differential equations, cannot be easily incorporated into standard examinations and may therefore be neglected or shunned in favor of more exam-friendly topics. Another alternative, particularly relevant to numerical problems, would be to set individual mini projects worth more marks than the usual shorter problem-based assignments. Such mini projects could also be a convenient vehicle for students to gain familiarity with the usage of computer - algebra or simulation software packages.

7. Conclusions

The recent literature on engineering-mathematics education reveals that there are many challenges ahead for the teaching of the subject. The increasing numbers of people who now have access to higher education, and the consequent wide range of abilities, backgrounds and motivations of those taking service courses in mathematics, presents some unique problems to educators. It is clear that there are no quick fixes or easy solutions to these problems. It is also apparent that mathematics courses need to be sensitive to the needs of the engineering courses they are serving. Certain key skills and concepts need to be taught as early as possible and emphasized as often as possible. There is also a need to anticipate, wherever possible, which areas or topics need to be taught before others.

The continuous development of transferable and general mathematical modeling skills is an extremely important aspect of engineering mathematics courses, and should as a general principle take precedence over the learning of specific techniques that may themselves become less relevant with time as the result of technological progress.

Taking into consideration the range of abilities and levels of interest students are likely to bring when they first start service mathematics courses, encouragement for the use of more programmed self-study material, together with a more imaginative and less formal approach on the part of teachers, will be more effective in the long run.

It is evident that computer-algebra and other types of software, such as simulation software, will play an increasingly important rôle in the way engineering mathematics is taught. Online resources will continue to develop and offer greater possibilities for both teaching and self-study. Indeed, it is safe to say that as a result of the digital revolution, we are now on the cusp of a paradigm shift in terms of how engineering mathematics is taught and how students will learn the subject.

Finally, through effective teaching of the subject, mathematics should be seen by prospective engineers as something that is directly relevant, useful and indeed essential to engineering rather than a complicating hindrance. If this can be achieved, even if just partially, then mathematics and engineering will continue to have their productive and symbiotic relationship. The inspirational examples set by historical figures such as Galileo, James Clerk Maxwell, as well as all the other great thinkers over the ages who have managed to successfully combine engineering acumen, technical expertise and mathematical insight, all give testimony to this truth.

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